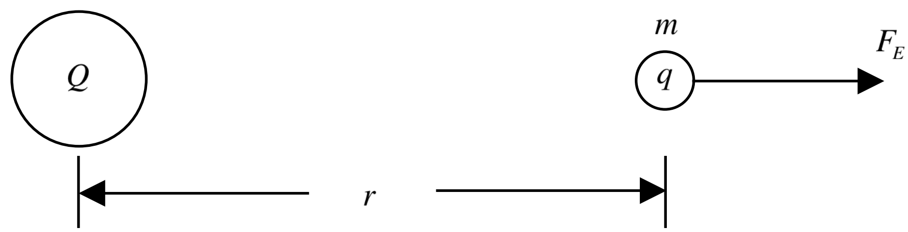
Motion of Charged Particles in Electric Fields

We will be looking at two situations in which a charged particle is moving in an electric field. The first situation occurs when the motion of the charged particle is due to the presence of a fixed charged particle. The other analysis will involve the motion of a charged particle through a uniform electric field, such as that between two parallel plates.

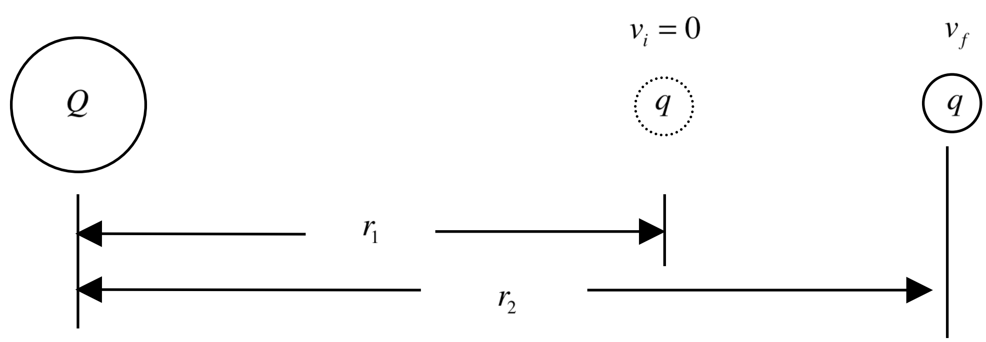
Part A – Motion of a charged particle due to the presence of another charged particle

Consider a small positive charge q, with a very small mass m, a distance r from a fixed positive charge Q. We will assume that m is so small that gravitational effects can be ignored.



Describing the object's motion in terms of dynamics is difficult. As it begins to move, r increases, causing F_E to decrease, so that a decreases as well. Motion with changing acceleration is difficult to analyze using dynamics.

If we use conservation of energy to analyze its motion, it becomes much simpler. As the separation between the charges increases, the electric potential energy decreases and the charge q begins to acquire kinetic energy. The Law of Conservation of Energy states that the total energy remains constant.



At r_1

$$E = PE_E = +\frac{kQq}{r_1}$$

At r_2

$$E = PE_E + E_k$$
$$= +\frac{kQq}{r_2} + \frac{1}{2}mv_f^2$$

Therefore,

$$\boxed{\frac{kQq}{r_1} = \frac{kQq}{r_2} + \frac{1}{2}mv_f^2}$$

$$-\left[\frac{kQq}{r_2} - \frac{kQq}{r_1}\right] = \frac{1}{2}mv_f^2$$
$$-\Delta PE_E = \Delta E_k$$

Where,

r_1 is the initial separation between the two charges

r_2 is the separation after being accelerated (gaining E_k)

v_f is the final speed of the particle

Example 1

A small pith ball of mass $1.0\times10^{-5}\ kg$ and charge $+2.0\times10^{-9}\ C$ is at rest $25\ cm$ from a fixed positive charge of $5.0\times10^{-6}\ C$. Neglecting gravitational effects and air resistance, with what speed will the pith ball be moving when it is $50\ cm$ from the other charge?

Part B – Motion of a charged particle in a uniform electric field

When the electric field in which the charged particle is moving is a uniform electric field, its motion is much simpler. In a uniform electric field

$$F_E = qE = \text{constant}$$

Therefore,

$$a = \frac{F_E}{m} = \text{constant}$$

Thus, the charged particle moves with constant acceleration, allowing us to analyze its motion using dynamics. This will be the case for small particles (such as protons, electrons, and ions) moving between parallel plates.

In the case of parallel plates, it is often desirable to analyze the motion in terms of conservation of energy. In a parallel plate apparatus, whose separation is d, the work done in moving a charge q from one plate to the other is

$$\begin{aligned} W &= F_E \cdot d \\ &= qEd \\ &= q \cdot \frac{\Delta V}{d} \cdot d \end{aligned}$$

$$\boxed{W = q\Delta V}$$

This amount of work is equal in magnitude to the change in kinetic energy of the particle as it moves from one plate to the other.

Example 2
The potential difference between two parallel plates is $8000\ V$. If a free electron of mass $9.1\times10^{-31}\ kg$ and charge $1.6\times10^{-19}\ C$, is released at the negative plate, with what speed does it hit the positive plate?

Charges in Electric Fields Worksheet #1

1. The potential difference between two parallel plates is $1.5 \times 10^2\ V$. If 0.24 J of work is required to move a small charge from one plate to the other, what is the magnitude of the charge? ($1.6 \times 10^{-3}\ C$)

2. An alpha particle has a positive charge of 2e and a mass of $6.6 \times 10^{-27}\ kg$. With what velocity would an alpha particle reach the negative plate of a parallel plate apparatus with a potential difference of $2.0 \times 10^3\ V$, if it started, at rest,
   a. next to the positive plate? ($4.4 \times 10^5\ m/s$)
   b. at the midpoint between the plates? ($3.1 \times 10^5\ m/s$)

3. A pith ball of mass $1.0 \times 10^{-5}\ kg$ with a positive charge of $4.0 \times 10^{-7}\ C$ is slowly pulled by a string a distance of 50 cm through a potential difference of $8.0 \times 10^2\ V$. It is then released from rest and "falls" back to its original position. Calculate:
   a. the work done by the string in moving the pith ball. ($3.2 \times 10^{-4}\ J$)
   b. the average force required to do this work. ($6.4 \times 10^{-4}\ N$)
   c. the kinetic energy with which the pith ball reaches its original position. ($3.2 \times 10^{-4}\ J$)
   d. its speed just as it reaches its original position. (8.0 m/s)

4. Two electrons are held, at rest, $10^{-12}\ m$ apart, and then released. With what kinetic energy and speed is each moving when they are a "large" distance apart? ($1.2 \times 10^{-16}\ J$, $1.6 \times 10^7\ m/s$)

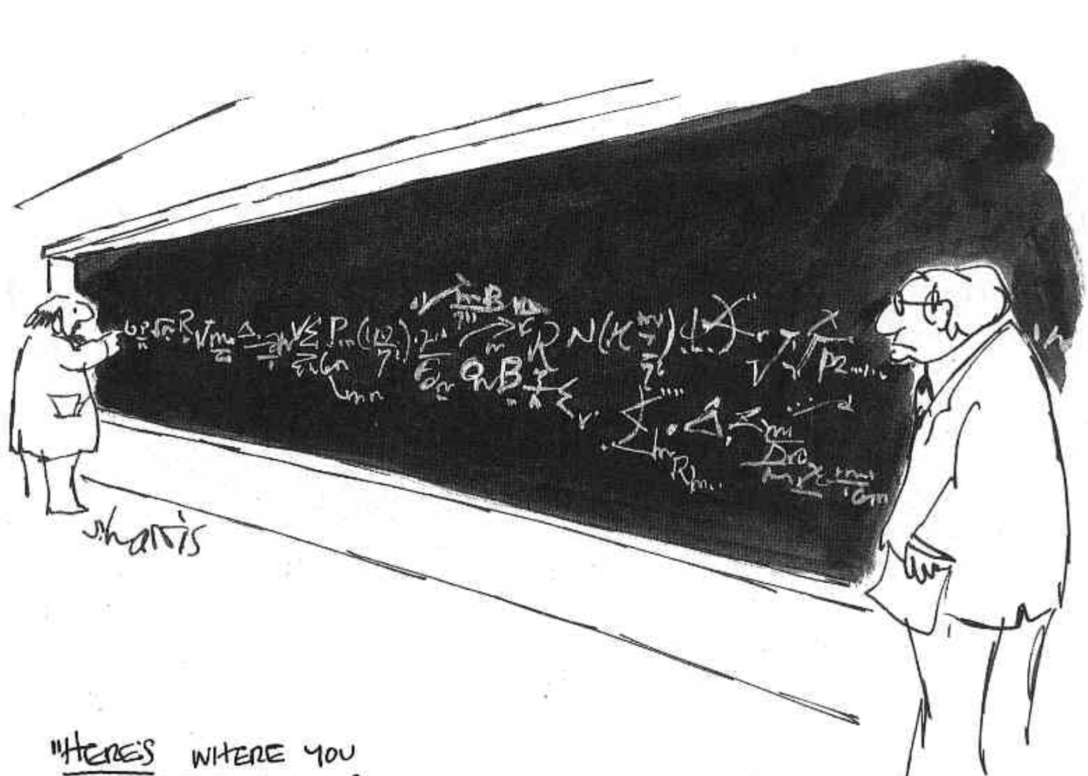
5. What potential difference is required to accelerate a deuteron, of mass $3.3 \times 10^{-27}\ kg$ and charge $1.6 \times 10^{-19}\ C$, from rest to a speed of $5.0 \times 10^6\ m/s$? ($2.6 \times 10^5\ V$)

6. An electron is accelerated by a constant electric field of 300 N/C.
   a. Find the acceleration of the electron. ($5.27 \times 10^{13}\ m/s^2$)
   b. Find the electron's speed after $1.0 \times 10^{-8}\ s$ assuming it starts from rest.($5.27 \times 10^5\ m/s$)

7. A constant electric field directed along the positive x-axis has a strength of $2.0 \times 10^3\ N/C$.
   a. Find the electric force exerted on a proton by the field. ($3.2 \times 10^{-16}\ N$ along the positive x-axis)
   b. Find the acceleration of the proton. ($1.9 \times 10^{11}\ m/s^2$)
   c. Find the time required for the proton to reach a speed of $1.0 \times 10^6\ m/s$, assuming it starts from rest. ($5.3 \times 10^{-6}\ s$)

8. Consider an electron that is released from rest in a uniform electric field.
   a. If the electron is accelerated to 1.0 percent of the speed of light after traveling 2.0 mm, what is the strength of the electric field? ($1.3 \times 10^4\ N/C$)
   b. What speed does the electron have after traveling 4.0 mm from rest? ($4.2 \times 10^6\ m/s$)

9. A proton has a kinetic energy of $3.25\times10^{-15}\ J$. What is the magnitude and direction of the electric field that will stop this proton in a distance of 1.25 m? ($1.62\times10^{4}\ N/C$ in the direction opposite to the proton's velocity)



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